Lesson 5-5: Inequalities in Triangles

Some common sense stuff

Today's lesson is going to formalize some properties of triangles that I'll bet you will see as intuitively obvious. In the past we've talked about some special triangles: isosceles and equilateral for instance. If I gave you an isosceles triangle, and told you which sides are congruent, you would be able to identify which angles are congruent. It would be the angles opposite the congruent sides.

But, what if I gave you a scalene triangle? The sides of the scalene triangle all have different measures. If I told you which side was longest, could you pick the largest angle in the triangle?

Properties of inequality

Before we get rolling, we need to refresh our memories about how inequalities work. Just as there are properties of equality and of congruence, there are similar properties for inequality. Here they are in a table (we will use the abbreviation POI for these):

Properties of Inequality	**For all real numbers *a, b, c,* and *d*:**
Addition POI	If $a > b$ and $c \geq d$, then $a + c > b + d$
Multiplication POI	If $a > b$ and $c > 0$, then $ac > bc$
	If $a > b$ and $c < 0$, then $ac < bc$
Transitive POI	If $a > b$ and $b > c$, then $a > c$
Comparison POI	If $a = b + c$ and $c > 0$, then $a > b$

Of particular importance for our discussion about inequalities in triangles is the comparison property of inequality. If you are having a hard time getting your arms around this one, think of the example given in the book. Suppose I have a container of juice and I pour ½ (or as close as I can) into one glass and the other ½ into another glass. It would be pretty tough to pour them so exactly ½ is in each glass. But, I **can** be absolutely positive that each glass holds **less** than the original container did. That really is all the comparison POI is saying.

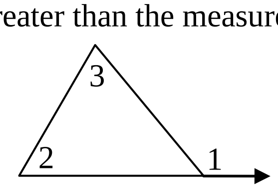
Applying this to triangles…

Do you remember the Triangle Exterior Angle Theorem? It states that the measure of each exterior angle of a triangle is equal to the sum of the measures of its two remote interior angles. There is a corollary to this if you apply the comparison POI:

Corollary to the Triangle Exterior Angle Theorem

The measure of an exterior angle of a triangle is greater than the measure of each of its remote interior angles.

$m\angle 1 > m\angle 2$ and $m\angle 1 > m\angle 3$



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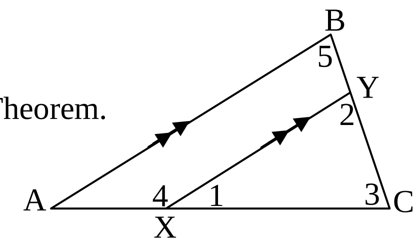
If that seems obvious to you, well it should. Most corollaries are things that if you just take a second and think about them, are obvious.

Example – not in the book

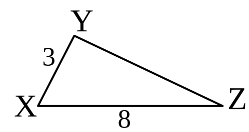
Explain why $m\angle 4 > m\angle 5$.



$m\angle 4 > m\angle 2$ by the Corollary to the Exterior Angle Theorem.
$m\angle 2 = m\angle 5$ since $\overline{AB} \parallel \overline{XY}$ & they're corr. $\angle$'s.
$m\angle 4 > m\angle 5$ by subst. POI
Q.E.D.

A conjecture about angle sizes…

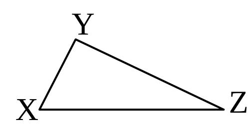
Consider the scalene triangle to the right. Based on the information given ($XZ > XY$) how would you say that $\angle Y$ compared with $\angle Z$?



An obvious conjecture is that because XZ is so much bigger than XY the angle opposite XZ must be bigger…the sides are wider apart. This is indeed the case.

Theorem 5-10

If two sides of a triangle are not congruent, then the larger angle lies opposite the longer side.



If $XZ > XY$ then $m\angle Y > m\angle Z$.

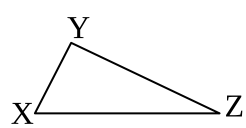
The converse is also true…

Theorem 5-11

If two angles of a triangle are not congruent, then the larger side lies opposite the larger angle.

If $m\angle Y > m\angle Z$ then $XZ > XY$.

Let's prove this using indirect reasoning:
Given: $m\angle Y > m\angle Z$
Prove: $XZ > XY$



Proof: Assume the negative of what we want to prove: $XZ \leq XY$
Here we have two cases: 1) $XZ < XY$ and 2) $XZ = XY$…consider each by itself.

If $XZ < XY$ then $m\angle Y < m\angle Z$ by Theorem 5-10.
This contradicts the given that $m\angle Y > m\angle Z$. Therefore $XZ < XY$ must be false.

If $XZ = XY$ then $m\angle Y = m\angle Z$ by Isosceles Triangle Theorem.
This contracts the given that $m\angle Y > m\angle Z$. Therefore $XZ = XY$ must be false.

Therefore, the assumption that $XZ \leq XY$ is false, so $XZ > XY$.
Q.E.D.

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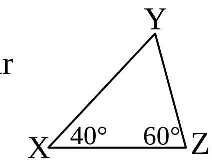
Example – Pg 275 Check Understanding 3

List the sides of the ΔXYZ in order from shortest to longest. Explain your listing.



$m\angle Y = 80$ thus $YZ < XY < XZ$

A conjecture about triangle side length

Can you form a triangle out of any three segments? What about the following three segments: 4cm, 4cm, 6cm? You should quickly see that you can form a triangle from them.

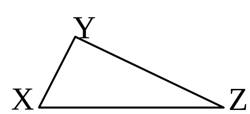
How about these: 4cm, 4cm, 10cm? They need to meet endpoint-to-endpoint. Can you do it? Nope, you can't. That third leg is just too long.

Looking at those two examples, can you form a conjecture about the lengths of segments that will form a triangle?

Theorem 5-12: The Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

$XY + YZ > XZ$
$YZ + XZ > XY$
$XZ + XY > YZ$



Example – Pg 276 Check Understanding 4

Can a triangle have the given lengths? Explain.

a) 2m, 7m, and 9m
   no; $2+7$ is not greater than 9

b) 4yd, 6yd, and 9yd
   yes; $4+6>9; 6+9>4; 4+9>6$

Example – Pg 276 Check Understanding 5

A triangle has sides of lengths 3in. and 12in. Describe the lengths possible for the third side.

12 + 3 = 15 so the 3rd side must be less than 15.
12 – 3 = 9 so the 3rd side must be greater than 9.
Therefore $9 < x < 15$

Homework

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